Progression in written methods

Progression in addition

Early stages

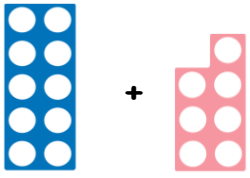
Children will engage in a wide variety of songs and rhymes, games and activities. They will begin to relate addition to combining two groups of objects, first by counting all and then by counting on from the largest number.

In practical activities, images and through discussion they will begin to use the vocabulary involved in addition:

'You have five apples and I have three apples. How many apples altogether?'

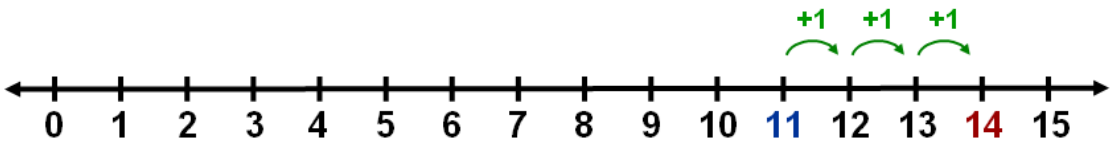
They can use a number line to count on for addition, counting on from the largest number e.g. 'Put your finger on number five. Count on four.'

Children will use other visuals to support their understanding of addition e.g. Numicon and Base 10



Children will then progress to a **marked number line:**

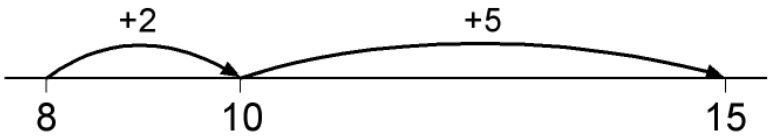
11 + 3 = 14



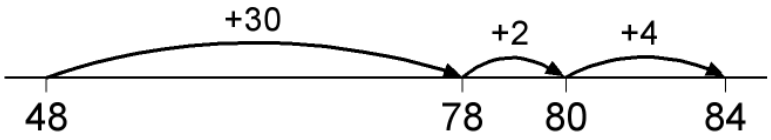
The empty number line method

The empty number line method helps to record the steps on the way to calculating the total. The biggest number is written on the left and the smaller number is added on using partitioning. Once secure, it can be used in any year group.

8 + 7 = 15



48 + 36 = 84

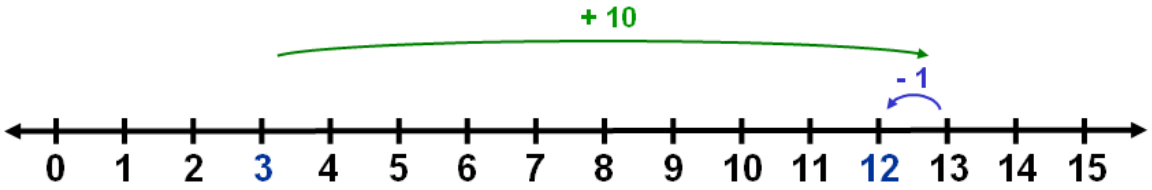


Compensation (applicable to all levels):

The compensation strategy for addition is useful when one of the numbers ends in 8 or 9. One number is rounded up to the next 10, the addition is carried out and then the answer is adjusted to compensate for the original change. For example:

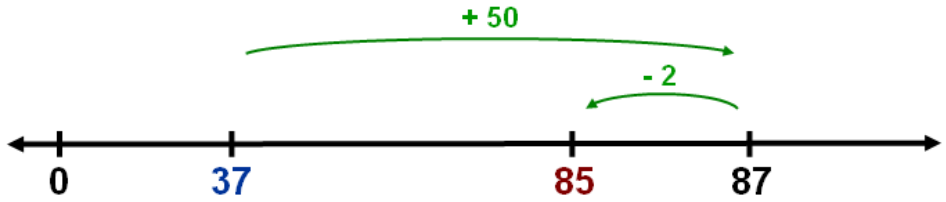
u u

3 + 9 =



t u t u

37 + 48



Partitioning method – an effective mental strategy

Addition is taught by partitioning numbers into tens and units before adding them, extending to hundreds, tens and units. The calculations are set out horizontally. Children need to secure a good understanding of place value.

For example:

65 + 49 =

60 + 40 = 100

5 + 9 = 14

100 + 14 = 114 ← Add the partial products (tens and units)

or

47 + 76 =

(47 + 70) + 6 = 117 + 6 = 123

Note: Pupils should still have practical equipment available to them as they move towards more formal methods.

Expanded column addition

6 3
3 2
5 (3 + 2)
9 0 (60 + 30)
9 5

Add the units together first and then the tens in preparation for the formal written method.

This leads to formal column addition

- The formal written method is taught as column addition using the 'carrying' method. In this common method, the carrying digits are recorded below the line, using the words 'carry ten' or 'carry one hundred', not 'carry one'.
- Accurate use of the language of place value when carrying will support pupil progression and understanding.

	h	t	u
	2	7	9
+	3	8	3
	6	6	2
	1	1	

	Th	h	t	u
	5	3	4	7
		2	8	1
+			1	7
	5	6	4	5
		1	1	

This method is efficient when used with larger whole numbers, money and decimals. Once learned, this method is quick and reliable.
Similar principles to those above can be applied to **money and decimals**.

	u	$\frac{1}{10}$	$\frac{1}{100}$
	7	. 4	4
+	2	. 5	8
	10	. 0	2
	1		1

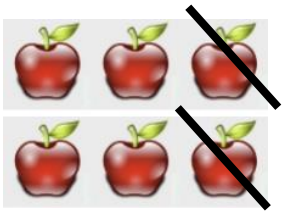
Progression in subtraction

Early stages

Children will engage in a variety of counting songs, rhymes and practical activities. In practical activities and through discussion they will begin to use the vocabulary associated with subtraction.

They will find one less than a given number.
They will begin to relate subtraction to 'taking away' using objects to count 'how many are left' after some have been taken away.

6 – 2 = 4

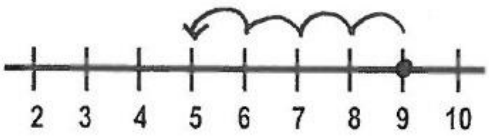


'Take two apples away. How many are left?'

Children will begin to count back from a given number.

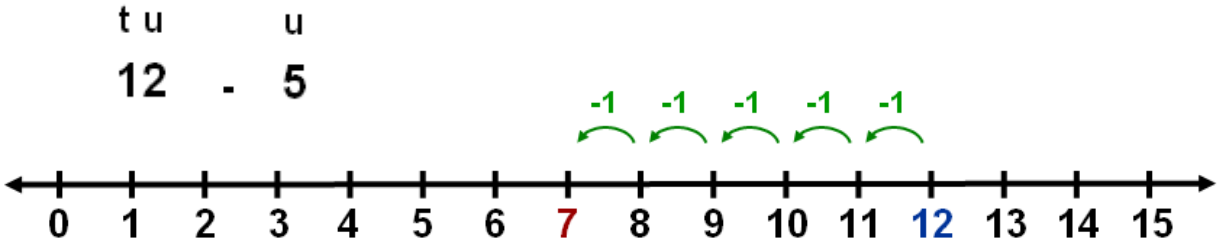
Children will then use a marked number line to count back for subtraction:

9 – 4 = 5

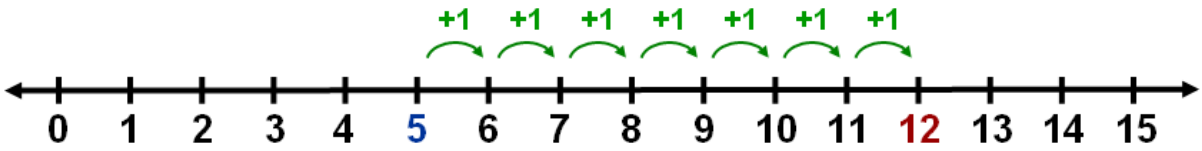


'Put your finger on number nine. Count back four.'

Counting back or forward in ones



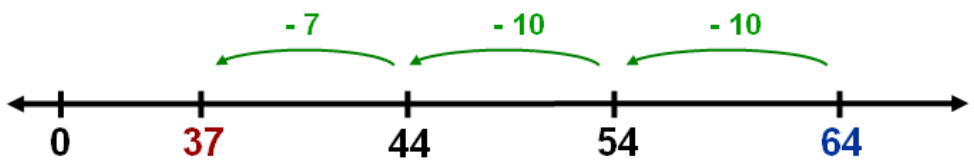
Counting on – what is the difference between 12 and 5?



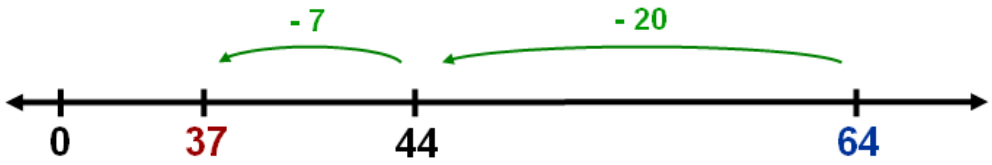
The empty number line - Counting back in tens

The larger number is written on the right hand side of the number line. The smaller number is subtracted by partitioning it into tens and units. Tens are subtracted first and then the units:

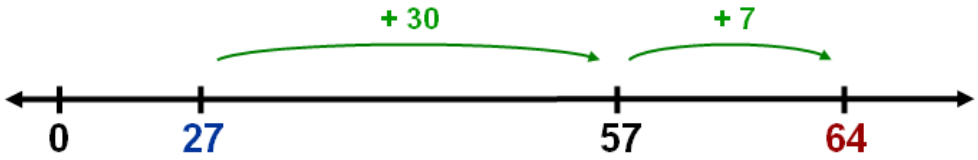
64 – 27 = 37



If children are confident, they are encouraged to use more efficient jumps:



Children could also count on using complimentary addition:



Counting on is a particularly effective strategy if two numbers are close together and children will need to use this as a mental method in KS2.

For example, it would be inefficient for a pupil to use 'taking away' on a number line to calculate 91 – 85 as counting on is quicker.

This leads onto partitioning:

t u t u
61 - 27

= 61 - 20 - 7

= 41 - 7

= 34

<u>Formal written method – decomposition or exchanging</u>

If the number in one column to be subtracted is larger than the top number, then a ten/hundred is 'exchanged' from the left and carried over to the right.

Use the term 'exchanged' and not 'borrowed' as you are not 'borrowing' anything!

t u	h t u	h t u
$\not{5}^{4}\,\not{4}^{1}$	$\not{5}^{4}\,\not{2}^{1}6$	$\not{5}^{4}\,\not{0}^{9}\,\not{0}^{1}$
- 1 6	- 3 7 4	- 3 7 4
3 8	1 5 2	1 2 6

When children are confident, develop with four digit numbers:

Continue to practise and apply the formal written method with large numbers, money and decimals.

u	$\frac{1}{10}$	$\frac{1}{100}$
$\not{7}^{6}$	. $\not{4}^{13}$	4^{1}
- 2	. 5	8
4	. 8	6

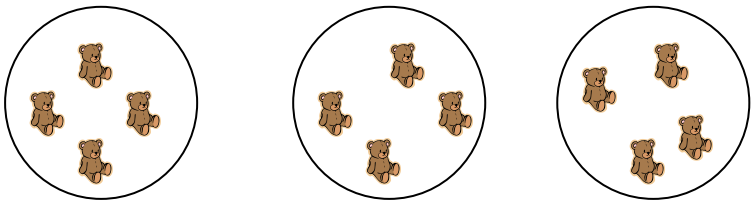
Progression in multiplication

Early stages

Children will engage in a wide variety of songs and rhymes, games and activities. In practical activities and through discussion, they will begin to solve problems involving doubling e.g. 'Three apples for you and three apples for me. How many apples altogether?'

Repeated addition – regular increments

Multiplication is taught as repeated addition to begin with: Children will count repeated groups of the same size in practical contexts to begin with, before moving to pencil and paper recording.



4 + 4 + 4 = 12

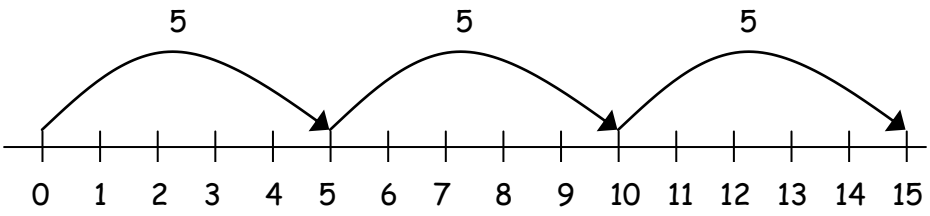
3 groups of 4 = 12

3 x 4 = 12

Number lines

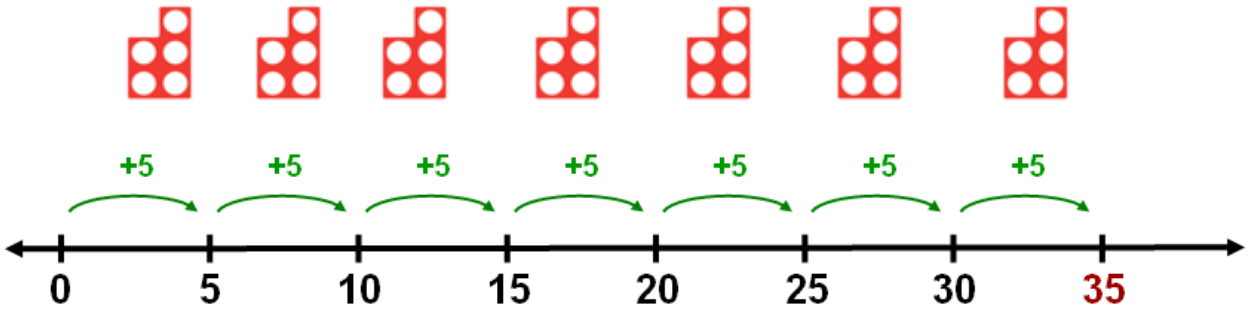
Repeated addition will then be taught on a number line:

3 x 5 = 5 + 5 + 5

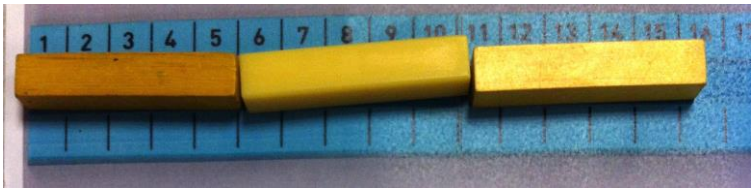


This can also be represented visually using Numicon:

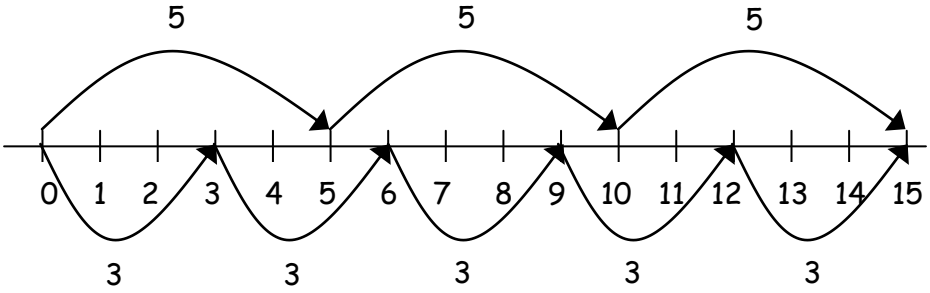
5 x 7 =



Repeated addition can also be modelled on a counting stick:

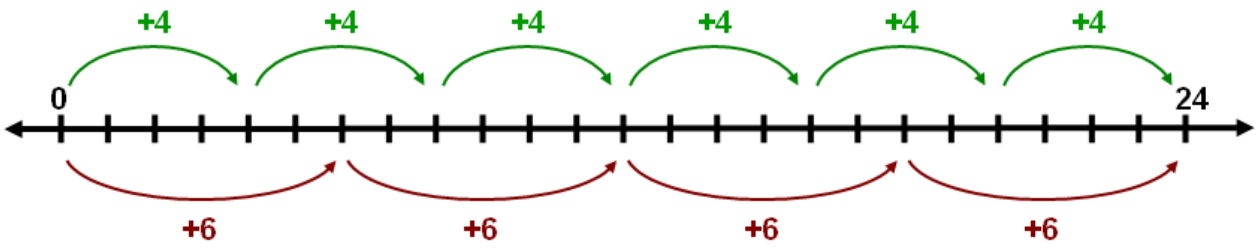


Commutativity -
Children should know that 3 x 5 has the same answer as 5 x 3. This should be taught on a number line so children can visually see the relationship:



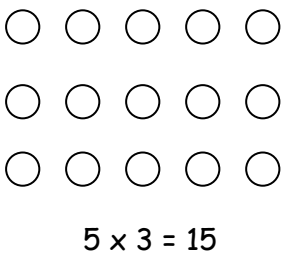
5 x 3 = 3 x 5

Empty number lines: 6 x 4 = 4 x 6



Arrays – visualizing the commutative law

Children should be able to model a multiplication calculation using an array:



3 x 5 = 15

5 x 3 = 15

'5 + 5 + 5 = 15'
'3 rows of 5'
'5 groups of 3'
'3 groups of 5'
'5 x 3 = 15'
'3 x 5 = 15'

Partitioning Method
Multiply the tens and then the units and recombine, for example:

4 x 24 =
4 x 20 = 80
4 x 4 = 16
80 + 16 = 96

Extending to HTU x U

This strategy is needed in Key Stage 2 as a quick mental method.

Expanded short multiplication:

36 x 4 = 144

```
 T   U
30 + 6
 x   4
   2 4 (4 x 6)
+ 1 2 0 (4 X 30)
  1 4 4
```

Expanded with partial products to understand the importance of place value

Refine the recording in preparation for formal short multiplication:

```
   T U
     3 6
 x   4
     2 4 (4 x 6)
+ 1 2 0 (4 x 30)
   1 4 4
```

Use the language of place value to ensure understanding.

Formal short multiplication: Compact method

```
    36
x   4
 1 4 4
   2
```

- Work RIGHT to LEFT – units first
- Ensure that the digit 'carried over' is written under the line in the correct column.

If children are confident, continue to develop short multiplication with 3 and then 4 - digit numbers multiplied by a one-digit number:

127 x 6 = 762

```
    1 2 7
      x 6
      4 2 (6x7)
+   1 2 0 (6x20)
    6 0 0 (6x100)
    7 6 2
```

This will lead into the short formal method:

1 2 7
x 6
7 6 2
1 4

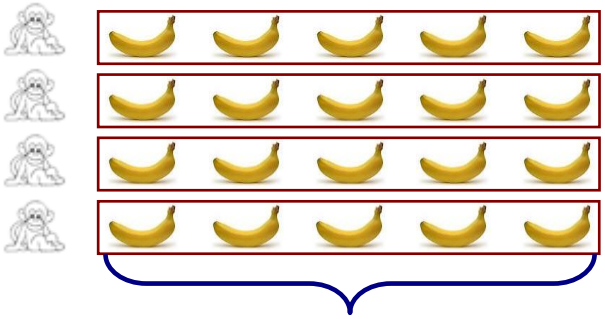
Progression in division

Equal sharing

To begin with division is taught as sharing, children use practical and informal methods to solve simple problems.

20 bananas need to be shared equally between **4** monkeys.
How many does each monkey get?

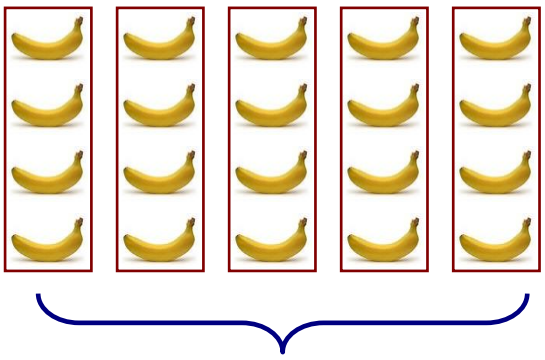
20 ÷ 4 =



Each monkey gets 5 bananas

Equal grouping

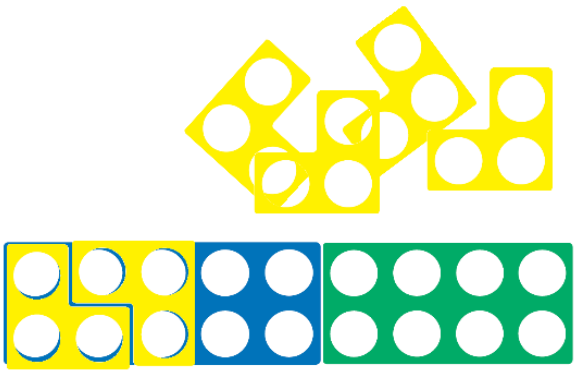
How many bunches (groups) of **4** bananas are there in a pile of **20**?
How many 4s in 20?



5 groups of 4 bananas make 20.

Children will use various models and images e.g. Numicon to support their understanding of division e.g. **18 ÷ 3 =**

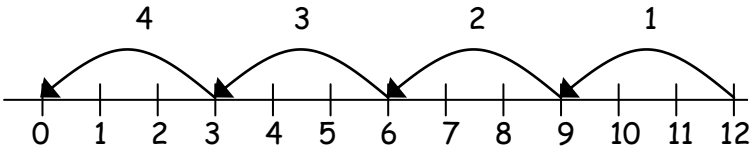
Covering the 18 with the '3' pieces - how many make 18?



Repeated subtraction/grouping using a number line

Children will then start to use a marked number line to support their calculation:

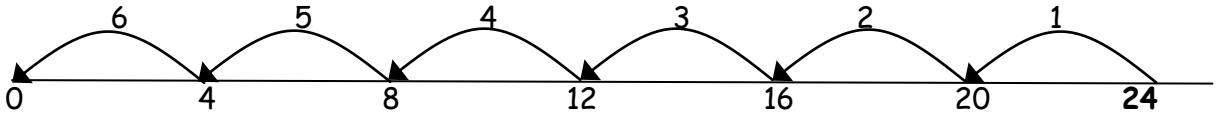
For example: 12 ÷ 3 = 4



Starting at the largest number and subtracting groups of 3 until zero is reached.

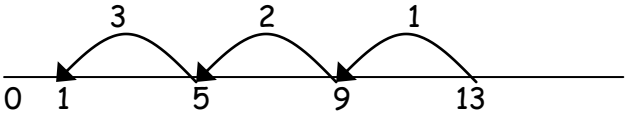
Progressing onto using an empty number line

24 ÷ 4 = 6



Children will also experience calculations involving remainders:

13 ÷ 4 = 3 r1



Take away groups of 4 until you are as close as possible to zero and cannot take away another group of 4. What is left is the remainder.

Short division – written method

Children are first taught to divide a two-digit number by a one-digit number, including division with remainders.

81 ÷ 3 =

$$3\overline{)8^{2}1}\quad 27$$

The carry digit '2' represents the 2 tens that have been exchanged for 20 ones. The 27 written above the line represents the answer.
The children then move on to use the same method as above to solve HTU divided by U and ThHTU divided by a U.

If you cannot divide a number by the first digit of the calculation, the following steps are applied:

$$4\overline{)368}$$

How many 4s go into 3?
Can't do…

$$4\overline{)368}$$

Underline 2-digits and ask how many 4s go into 36?

Children learn to record the remainder in different ways:

$$3\overline{)5^{2}4\ 4^{1}9}\quad 1816\ \text{r}\ 1$$

Using 'r' to represent the remainder

$$6\overline{)19^{1}6}\quad 32\ \tfrac{4}{6}$$

The remainder can also be expressed as a fraction (the remainder divided by the divisor)

$$3\overline{)5^{2}4\ 4^{1}9.0^{1}0\ 0}\quad 1816.3\dot{3}$$

Expressing the remainder as a decimal fraction. The dot on the last 3 in this example shows a reoccurring decimal answer.

Children need to be able to decide what to do after division and round up or down accordingly. For example 62 ÷ 8 is 7 remainder 6, but whether the answer should be rounded up to 8 or rounded down to 7 depends on the context:

I have 62p. Sweets are 8p each. How many can I buy?
Answer: 7 (the remaining 6p is not enough to buy another sweet)

Apples are packed into boxes of 8. There are 62 apples. How many boxes are needed? Answer: 8 (the remaining 6 apples still need to be placed into a box)